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Letter to the Editor

Modal frequency deviations in estimating ring gear modes using smooth ring solutions

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1. Introduction

Ring gear structural modes of planetary gear sets employed in modern automotive, aerospace, marine and other industrial drive train systems often contribute significantly to the severity of the gear whine problem caused by transmission error excitation. This is because the dynamic forces generated at the planet-ring gear meshes can be transmitted into the casing and other attachments through the ring structures that form one of the major vibratory energy paths. This is made worse by the intended high-speed and high-load applications of this class of geared systems. Hence, understanding the nature of these modes and their effects on vibration transmissibility is crucial to the ability to tackle the acoustic noise concern successfully. However, analytical modelling of ring gear dynamics is not trivial due to the geometrical complexity of the tooth form. Furthermore, the ring gear modes are more complex and less conforming. A potentially simpler approach is to utilize analytical and/or computational solutions of smooth ring structures having nearly the same nominal dimensions but without the explicit presence of the spline and tooth geometries. This less complicated representation may provide a more effective way of studying the basic modal phenomenon and computing the critical response parameters that control gear vibration and ultimately gear whine.

In order to determine the feasibility of the above-mentioned hypothesis, this communication compares the modal frequencies of ring gears and idealized smooth rings, and quantifies the frequency deviations in applying the simpler smooth ring solutions to represent the primary modal behaviors of ring gear structures. The modal solutions of the equivalent smooth ring are obtained by applying the Kirkhope dynamic stiffness matrix formulation [1,2], while the modal frequencies of the actual ring gears of interest are computed using the dynamic finite element method (FEM). A series of parametric study is conducted to examine the effects of ring thickness

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and width dimensions on the levels of discrepancy observed. Specifically, the thickness to radius and width ratios denoted by t/r and t/w, respectively, are considered. Results from this comparative analysis will be scrutinized in detail to determine the feasibility of applying the existing smooth ring theories to predict natural modes and vibration transmissibility in planetary gear sets. Prior to discussing the analysis and results, a review of published work on planetary gear and circular ring vibrations is given below.

Most of published studies on planetary gear dynamics assume a rigid ring gear model with localized mesh stiffness [3–8]. Using this concept, detailed dynamic analyses have been carried out by considering numerous factors such as mesh stiffness, dynamic transmission error, frictional forces, manufacturing variances, assembly errors, torque fluctuations of the driving motor, and planet gear phasing. For instance, Kahraman [3] developed a purely torsional model to predict the natural frequencies of a single stage planetary gear train system to the level of accuracy normally required by gear designers. The author was able to deduce a closed form solution for the torsional natural frequencies in terms of a limited set of system parameters. The same author [4] also developed a non-linear time-varying dynamic model for systems with any number of planet gears to study the load sharing characteristics. Lin and Parker [5,6] studied the sensitivity of planetary gear natural frequencies and vibration modes to selected parameters, and subsequently examined the effects of unequally spaced planets on the vibration characteristics of a planetary system. Saada and Velex [7] studied the effects of mesh stiffness, helix angle, and ring support stiffness on the dynamic behavior of planetary gear trains, Velex and Flamand [8] extended this model to calculate the dynamic tooth loads on a planetary gear set.

Many of the proposed models were able to predict dynamic mesh forces and system modes quite sufficiently. However, the fundamental assumption of rigid ring gear renders the model incapable of predicting the extent of force and vibration transmissibilities through the ring gear structure, and quantifying the effects of ring gear modes. A search through the literature has resulted in only a small handful of articles that consider the effect of ring gear flexibility. In one of the papers, Ma and Botman [9] analyzed the dynamic loads that include the contribution from selected ring gear in-plane bending modes. Effects of tooth and spline geometries, and majority of the ring gear flexible modes were not examined. In another paper, Stolarski [10] considered only the quasi-static deformation of the ring assuming an oval shape function and applied the model to calculate the deviation in stresses due to moving pinions. His model does not account for the dynamics of the epicyclic gear train. This lack of earlier studies found makes it even more important to address the effects of ring gear flexible modes when attempting to improve the design of planetary gear trains.

In contrast, the literature contains an abundance of circular ring vibration work. The earliest analysis was performed by Love [11] that resulted in the estimation of the natural frequencies of the four basic classes of natural modes: (a) flexural vibration in the plane of the ring (radial inextensional), (b) flexural vibration out of the plane of the ring (out-of-plane bending), (c) torsional, and (d) extensional. Over the years, many researchers have re-worked this problem to study the effects of various parameters on the natural frequencies and mode shapes of circular rings with either circular or rectangular cross-sections. For examples Kirkhope [1,2] obtained a dynamic stiffness matrix for in-plane and out-of-plane motions that was applied to compute the natural frequencies and mode shapes. He included the parameters such as rotary inertia and transverse shear that were neglected in the classical theory developed by Love. Rao [12] calculated the effects of transverse shear and rotary inertia on the coupled twist-bending vibration modes of circular rings, whereas Bickford and Maganty [13] obtained the expressions for out-of-plane modal frequencies of a thick ring, which accounts for the variations in curvature across the cross-section. Charnley et al. [14] modified the equation of motion derived by Love for a general cross-section case, which included the rotary inertia effect. The wealth of research on circular ring vibrations coupled with the lack of work done related to flexible ring gears makes it very attractive to extend smooth ring theories to analyze ring gear dynamics, which is the premise for this note.

2. Modal analysis

A typical ring gear is essentially made up of a circular ring with tooth and spline features attached at certain regular intervals along the circumferential direction. These teeth and splines make the overall structure geometrically more complex and pose difficulty in applying the classical theory of continuum mechanics in order to obtain analytical expressions of natural frequencies, mode shapes and vibration transmissibility characteristics. The alternative approach is to employ the dynamic FEM that, although can often predict the modal quantities quite accurately, takes considerable effort and experience to apply. Furthermore, the FEM, due to its inherent purely computational nature, may not be able to provide the critical insights normally afforded by analytical treatments that can lead to a better understanding of the underlying physics of the problem. However, for our purpose here, the FEM can be applied to verify the accuracy and limitations of adopting smooth ring theories to investigate ring gear dynamics.

A dynamic finite element package [15] is applied in this study to perform the free vibration analysis of unconstrained ring gears and circular rings. The baseline geometries of these ring gear structures are shown in Table 1. To keep the model size small, the gear teeth are modelled as straight-faced instead of the exact involute shape and their root radii are not considered explicitly. Since the objective of this analysis is not to predict the modal quantities of specific gear problems, but rather to show the applicability of circular ring theories, these assumptions are not critical to the outcome of this study. The structures are discretized using a set of 10-noded tetrahedron elements. This type of higher order tetrahedron element is used to ensure a sufficiently accurate representation and to obtain a better approximation of the displacement field. Fig. 1 shows the finite element models of the two ring gear structures and the equivalent circular ring representations. All the modes between 0 and 25 kHz are extracted from the normal mode analysis based on the Lanczos scheme. Their natural frequencies for cases A and B are shown in

Table 1
Geometries of ring gear and circular ring structures of interest

	Outer diameter (mm)	Inner diameter (mm)	Width (mm)	Number of teeth	Number of splines
Ring gear A	201.14	184.94	34.29	114	17
Ring gear B	135.56	124.88	30.73	78	15
Smooth ring A	201.14	184.94	34.29	—	—
Smooth ring B	135.56	124.88	30.73	—	—

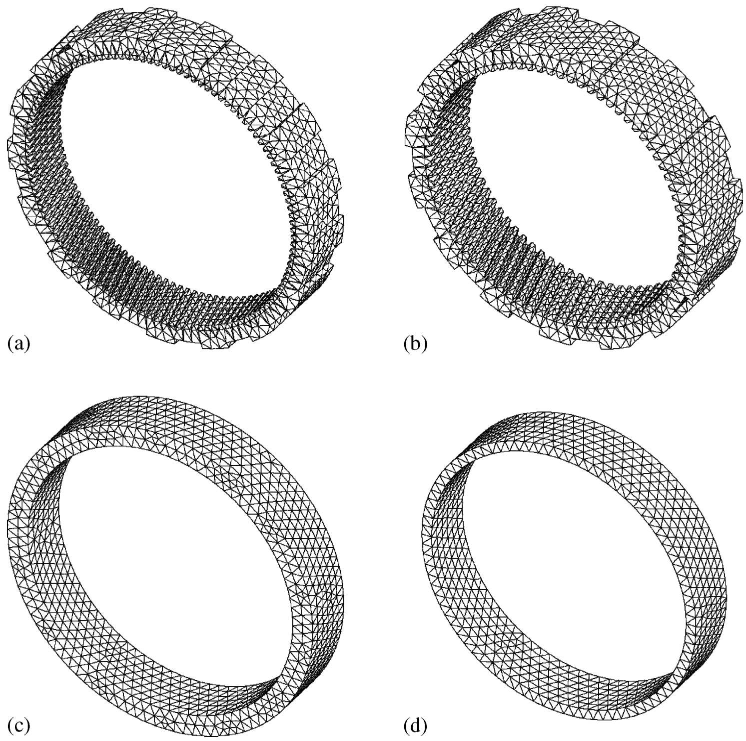

Fig. 1. Finite element models of the ring gear and smooth ring structures: (a) ring gear A, (b) ring gear B, (c) ring A, and (d) ring B.

Tables 2 and 3, respectively, along with the experimental data of ring gears and analytical results of smooth rings, which will be discussed more in detail later. The predicted mode shapes of the ring gears can be classified into (a) radial inextensional, (b) out-of-plane bending, (c) extensional, and (d) torsional classes of modes, as shown in Fig. 2. Within the same class, the individual modes are distinguished using the nodal diameters symbolized by n. Note that the radial inextensional and out-of-plane bending modes corresponding to $n = 0, 1$ are rigid body modes, and therefore are not illustrated in Fig. 2. Since the primary deformation of the radial inextensional modes is radial, they are expected to be the most efficient transmitter of the vibratory energy into the housing structure as their motions couple well with the transverse displacement of the external shell.

Kirkhope [1] provided a dynamic stiffness matrix with the effects of transverse shear and rotary inertia for in-plane vibration of thick circular ring. The formulation uses the Timoshenko shear coefficient that accounts for the variations in shear strain across the cross-section to represent the average rotation due to transverse shear. The resulting eigenvalue problem gives the natural frequencies and modes for in-plane flexure (radial inextensional), extensional, and transverse shear. Furthermore, Kirkhope [2] also derived a stiffness matrix for out-of-plane motion, which predominantly yields out-of-plane bending, torsion of the ring section with essentially no displacement of the centroidal axis, and transverse shear modes. The torsion constant used in the equation is given by Roark and Young [16]. The natural frequencies of the radial inextensional,

Table 2
Comparisons of the natural frequencies of ring gear A obtained from modal experiment and FEM, and smooth ring A obtained analytically and from FEM

n	Ring gear A (Hz)		Smooth ring A (Hz)		% deviation (I) vs. (II)	% deviation (III) vs. (IV)	% deviation(I) vs. (III)	% deviation(II) vs. (III)
	Expt. (I)	FEM (II)	Analytical (III)	FEM (IV)				
(a) *Radial inextensional*								
2	602	622	550	552	−3.32	−0.36	8.64	11.58
3	1691	1744	1547	1554	−3.13	−0.45	8.52	11.30
4	3207	3306	2942	2964	−3.09	−0.74	8.26	11.01
5	5113	5266	4710	4758	−2.99	−1.01	7.88	10.56
6	7379	7587	6827	6915	−2.82	−1.27	7.48	10.02
7	9945	10,216	9268	9413	−2.72	−1.54	6.81	9.28
8	12,734	13,009	12,008	12,226	−2.16	−1.78	5.70	7.69
9	16,344	16,728	15,021	15,330	−2.35	−2.02	8.09	10.20
(b) *Out-of-plane bending*								
2	1344	1312	1145	1167	2.38	−1.89	14.81	12.73
3	3961	3883	3497	3580	1.97	−2.32	11.71	9.94
4	7270	7133	6401	6631	1.88	−3.47	11.95	10.26
5	10,773	10,625	9331	9855	1.37	−5.32	13.39	12.18
6	14,305	14,077	12,084	13,081	1.59	−7.62	15.53	14.16
7	17,332	17,261	14,664	16,315	0.41	−10.12	15.39	15.05
(c) *Extensional*								
0	7699	7609	8482	8479	1.17	0.04	−10.17	−11.47
1	10,852	10,717	11,978	11,961	1.24	0.14	−10.38	−11.77
2	16,379	16,851	18,927	18,863	−2.88	0.34	−15.56	−12.32
(d) *Torsional*								
0	7230	6998	8449	8091	3.21	4.42	−16.86	−20.73
1	9027	8828	9416	9332	2.20	0.9	−4.31	−6.66
2	11,350	11,121	11,388	11,403	2.02	−0.13	−0.33	−2.4
3	14,211	13,982	14,216	14,348	1.61	−0.92	−0.04	−1.67
4	17,371	17,285	21,927	—	0.50	—	−26.23	−26.86

Table 3
Comparisons of the natural frequencies of ring gear B obtained from modal experiment and FEM, and smooth ring B obtained analytically and from FEM

n	Ring gear B		Smooth ring B (Hz)		% deviation (I) vs. (II)	% deviation (III) vs. (IV)	% deviation (I) vs. (III)	% deviation (II) vs. (III)
	Expt. (I)	FEM (II)	Analytical (III)	FEM (IV)				
(a) *Radial inextensional*								
2	910	940	797	802	−3.30	−0.62	12.42	15.21
3	2555	2634	2242	2263	−3.09	−0.93	12.25	14.88
4	4828	4985	4266	4320	−3.25	−1.25	11.64	14.42
5	7684	7924	6832	6942	−3.12	−1.58	11.09	13.78
6	11,031	11,370	9907	10,097	−3.07	−1.88	10.19	12.87
7	14,668	15,105	13,456	13,754	−2.98	−2.17	8.26	10.92
8	19,547	20,214	17,445	17,879	−3.41	−2.43	10.75	13.70
9	23,953	24,649	21,835	22,433	−2.91	−2.67	8.84	11.42
(b) *Out-of-plane bending*								
2	2129	2131	1680	1727	−0.09	−2.72	21.09	21.16
3	6035	6013	4805	5039	0.36	−4.64	20.38	20.09
4	10,570	10,476	8156	8825	0.89	−7.58	22.84	22.15
5	15,086	14,895	11,261	12,674	1.27	−11.15	25.35	24.40
6	19,234	18,973	16,895	16,624	1.36	1.63	12.16	10.95
7	22,367	22,043	19,571	20,772	1.45	−5.78	12.5	11.21
(c) *Extensional*								
0	10,094	10,835	12,574	12,565	−7.34	0.07	−24.57	−16.05
1	14,984	15,180	17,757	17,719	−1.31	0.21	−18.51	−16.98
2	23,488	23,689	28,060	—	−0.86	—	−19.47	−18.45
(d) *Torsional*								
0	9660	10,149	12,574	12,235	−5.06	2.77	−30.17	−23.89
1	12348	12,414	13,541	13,521	−0.53	0.15	−9.66	−9.08
2	15,770	15,889	16,828	16,951	−0.75	−0.73	−6.71	−5.91
3	20,254	20,479	21,916	22,174	−1.11	−1.16	−8.21	−7.02

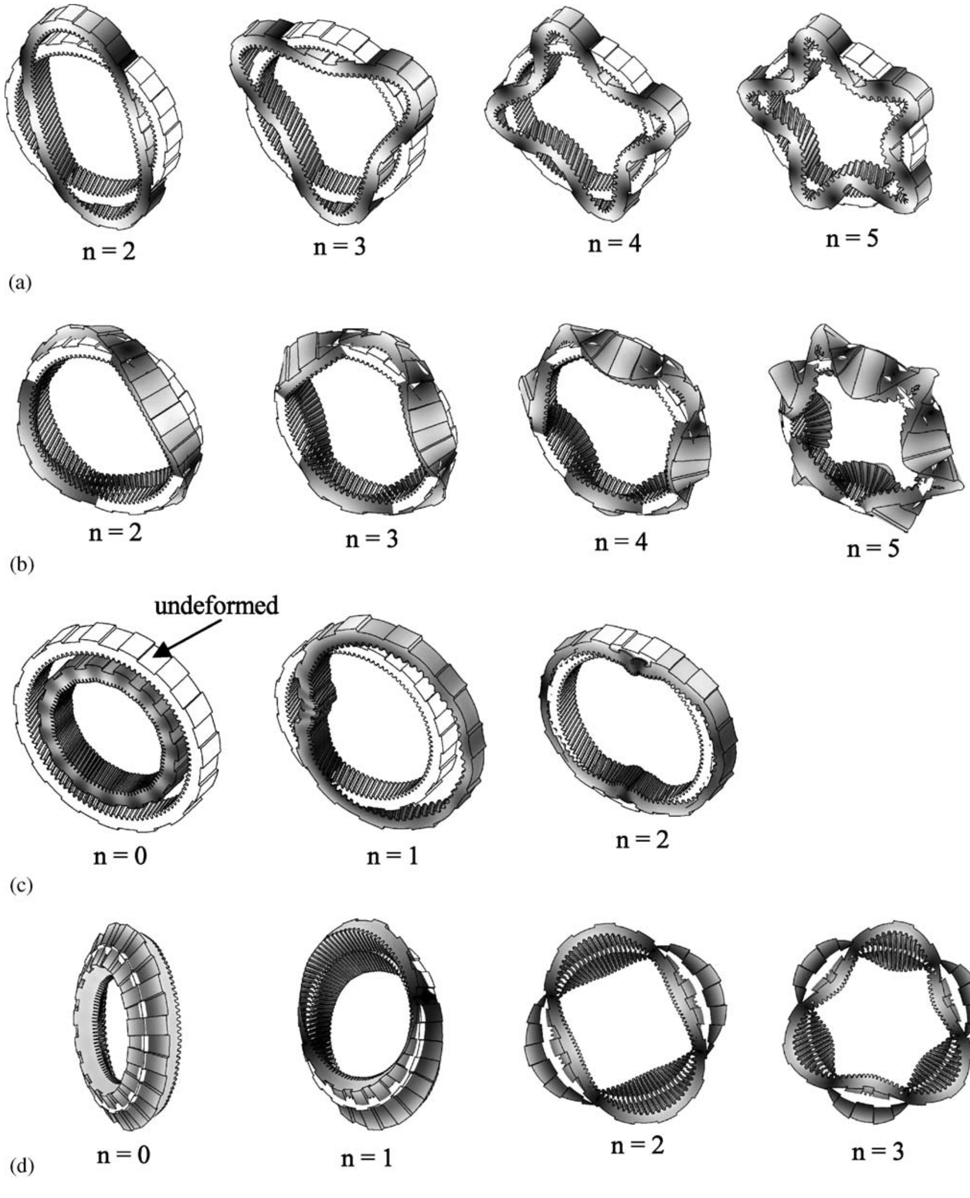

Fig. 2. Classification of ring gear structure modes in terms of n nodal diameters for each of the four classes of modes: (a) radial inextensional; (b) out-of-plane bending; (c) extensional; and (d) torsional.

extensional, out-of-plane bending and torsional classes of modes for the two smooth rings of interest applying the Kirkhope dynamic stiffness matrix formulations are also shown in Tables 2 and 3, respectively.

To provide further validation, the modal experiments depicted in Fig. 3 were carried out to obtain the natural frequencies and frequency response function (FRF) of the two ring gears defined in Table 1. In the experiment, the ring gears were suspended using a flexible cord to

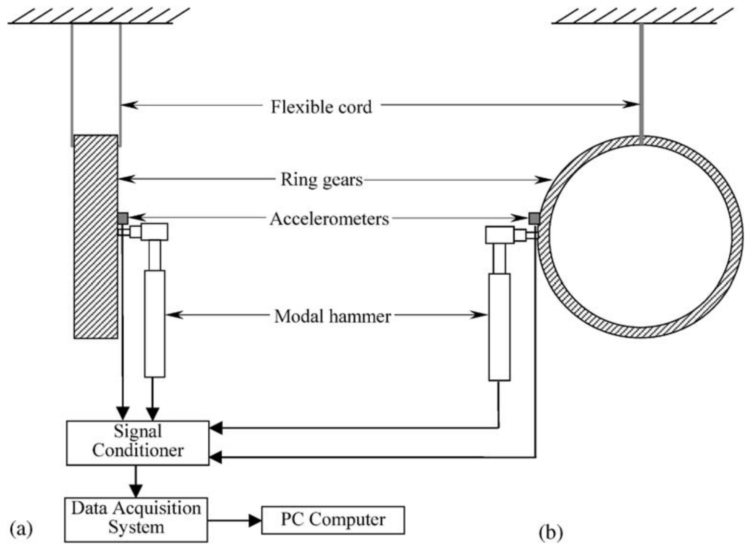

Fig. 3. Schematic of the modal experiment set-up: (a) normal excitation; and (b) in-plane excitation.

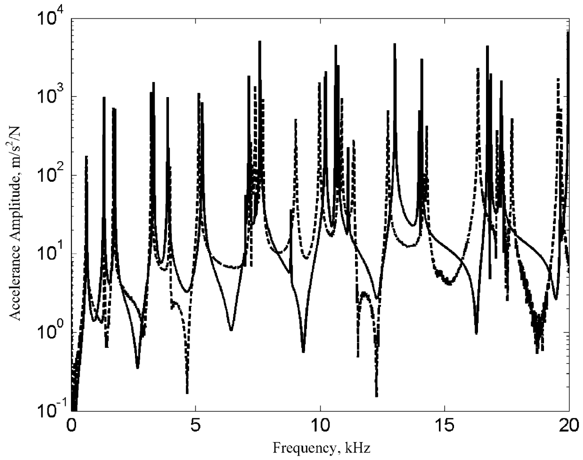

Fig. 4. Comparison of experimental (– – – –) and FEM predicted (——) driving point frequency accelerance response functions of ring gear A.

simulate the free boundary condition, and excited with a modal hammer (steel tip) in the in-plane and out-of-plane (normal) directions to increase the probability of finding all four classes of modes. Miniature accelerometers having up to 25 kHz of usable bandwidth were employed to acquire the vibration response. The data is then processed using a state-of-the-art 16-channel VXI data acquisition system to yield the FRF sought. Typical results (amplitude only) of the measured FRF are shown in Figs. 4 and 5 for ring gears A and B, respectively. From these FRF results, the

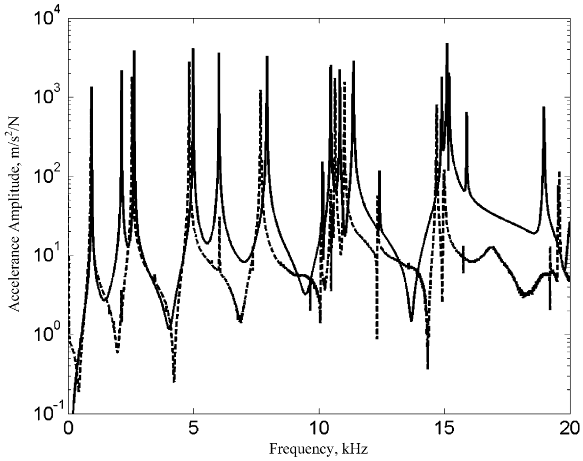

Fig. 5. Comparison of experimental (– – – –) and FEM predicted (——) driving point frequency accelerance response functions of ring gear B.

natural frequencies can be easily extracted. While identifying the natural frequencies, the phase angle was verified for consistency and the coherence was observed to be mostly in the range of 0.97–0.99. From Tables 2 and 3, the measured and predicted natural frequency results show excellent match for most modes. The slight deviation at higher frequencies may be attributed to the differences in the geometry and material property between the FEM model and actual hardware.

3. Results

Based on the natural frequencies obtained from the modal experiments, FEM and Kirkhope theory, tabulated in Tables 2 and 3, it is observed that many of the frequencies of the radial inextensional modes tend to fall within the typical frequency range of gear noise and vibration problem. Column 6 in Tables 2 and 3 shows the % deviation between experimentally obtained and FEM predicted natural frequency values of the ring gears. The deviations observed are less than 5% for most modes. Column 7 in Tables 2 and 3 shows the % deviation between analytical and FEM predicted smooth ring natural frequencies. Again, the deviations are less than 5% in most cases. Slight differences in both sets of results may be attributed to the differences in the geometry, material properties between actual hardware and FE model and uncertainty in measurement. These two results serve as the basis for the following comparisons.

Data in column 8 represents the % deviation between the measured natural frequency values of ring gears and analytical predictions of the corresponding equivalent smooth rings. For radial inextensional modes, these errors are generally 5–9% for case A, and 8–13% for case B. For out-of-plane bending modes, these errors are 11–26% for case A and 12–25% for case B. For extensional modes, the deviations become 18–25% for ring gear A and 10–16% for ring gear B. Lastly, for torsional modes, these frequency errors are 6–30% and 0–27% for cases A and B, respectively. On similar comparisons, the last column represents the % deviation between the

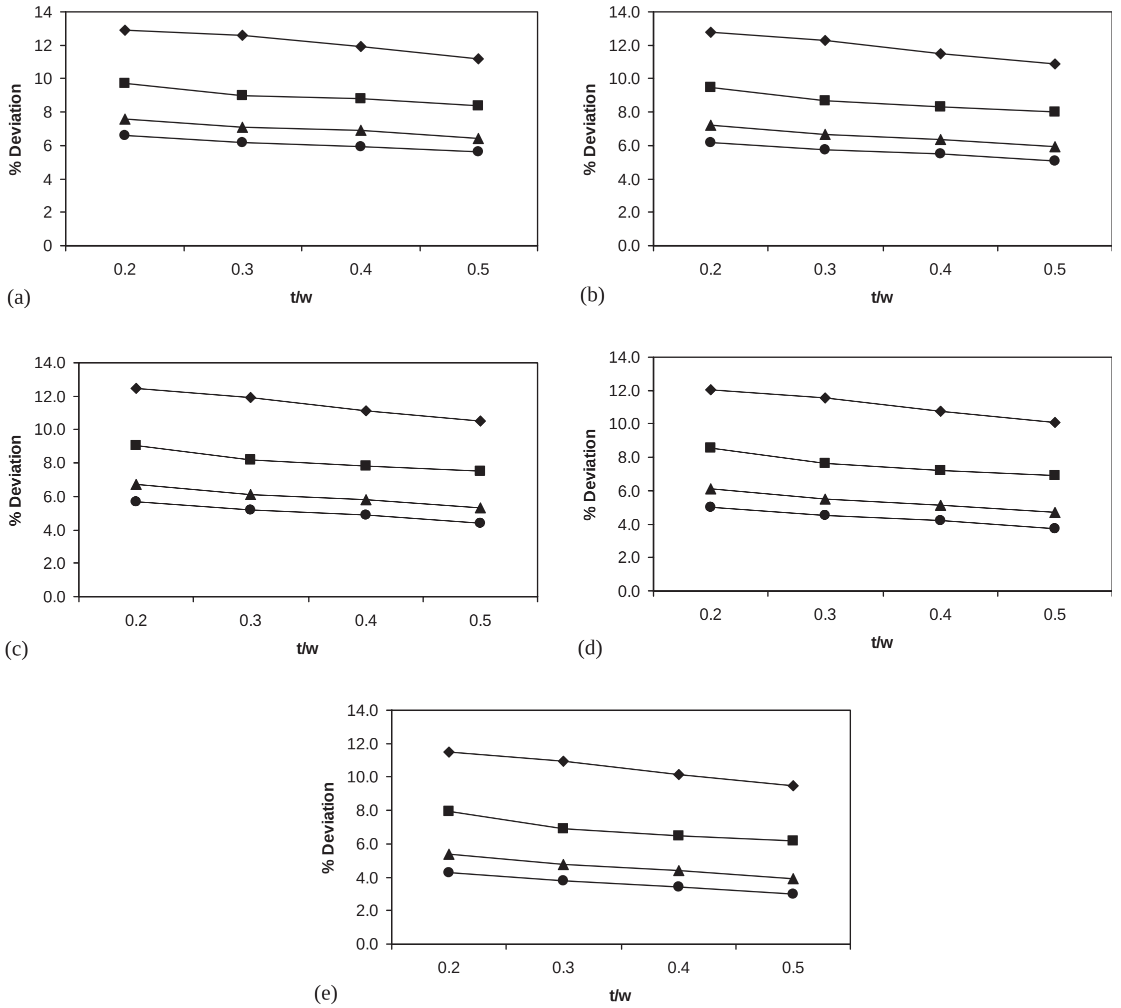

Fig. 6. Comparative effects of t/r and t/w on % deviation between ring gear (FEM) and ring (analytical) natural frequencies of radial inextensional modes: (a) $n = 2$; (b) $n = 3$; (c) $n = 4$; (d) $n = 5$; and (e) $n = 6$ (–◆–, $t/r = 0.07$; –■–, $t/r = 0.10$; –▲–, $t/r = 0.13$; –●–, $t/r = 0.15$).

natural frequency values obtained from the FEM model of the ring gears and analytical modal predictions of the smooth rings. For radial inextensional modes, the deviations are 10–16% for ring gear A and 7–12% for ring gear B. For out-of-plane bending modes, they are 11–25% for ring gear A and 9–15% for ring gear B. For extensional modes, these deviations are 16–19% for ring gear A and 11–13% for ring gear B, and finally for torsional modes, these deviations are 7–24% for ring gear A and 1–27% for ring gear B. For these cases studied, it can be observed that applying smooth ring results to model ring gears tend to underestimate the natural frequency value. The deviations seen for radial inextensional modes are approximately constant and do not change with n, whereas for other modes, the errors are sporadic in nature and do fluctuate considerably with n. This is probably because of the fact that radial inextensional modes, due to the nature of its deformation pattern, involve much lesser straining of teeth and splines than other classes of modes. Based on this finding, it can be suggested that the smooth ring theory may be used to predict the radial inextensional modes of ring gears quite accurately if the appropriate adjustments are applied. For other classes of modes, since the % deviation changes unpredictably with n, it becomes difficult to apply the modal results of regular rings to ring gear cases. Fig. 6 shows the deviations in radial inextensional natural frequencies (for $n = 2-6$) as a function of t/r and t/w. It is observed that the deviations are not constant, but change with thickness and width values. The deviations decrease slightly with increase in t/w, i.e., decrease in width. However, as t/r increases, these deviations decrease appreciably. Thus, for thicker ring gears, the discrepancies in applying smooth ring analytical equations is much lesser as compared to thinner ring gears. This is because as the thickness increases, the relative effect of teeth and splines becomes less significant and hence limiting the extent of frequency deviations.

4. Concluding remarks

From the comparative analysis shown, it can be concluded that splines and teeth in most cases have significant effect on the natural frequency values of ring gear structure. This renders the idea of applying smooth ring without teeth and splines less effective. However, if only radial inextensional modes are of interest, which is expected to be the most dominant mode affecting gear noise, then the ring gear may be modelled sufficiently as a smooth circular ring structure with minor adjustments. The tuning needed depends on the rim thickness and width of the ring geometry. In the case of gear whine, where most of the problematic frequencies lie within the first few modes of the radial inextensional class, the direct application of classical smooth ring theories, such as the ones suggested by Kirkhope, to more complex ring gear is feasible. For other classes of ring modes with the exception of the first two out-of-plane bending modes, the use of smooth ring theories produce large, irreparable discrepancies. The present findings are further being applied to examine the forced response problem. Also, the modification to the existing smooth ring theories to be most applicable to ring gears will be considered in future work.

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